TEMPERATURE FIELD OF AN INFINITE PLATE FOR VARIABLE VALUES OF THE HEAT TRANSFER COEFFICIENT AND THE TEMPERATURE OF THE EXTERNAL MEDIUM

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Equations are proposed for calculating the temperature field of an infinite plate in the stage corresponding to the regular regime of the second kind, when the heat transfer coefficient and the temperature of the external medium depend on time.

In connection with the temperature regime of plane elements under conditions of convective heat transfer the most important theoretical results have been obtained for the case of constant heat transfer coefficient. However, there are a number of cases in which this coefficient varies significantly with time. This applies to the formation of a thermal boundary layer under conditions of nonsteady flow over solid surfaces, heating in a pulsating flow, the temperature field of a ballistic body moving in a medium with variable density and temperature, etc.

The process of heat transfer in an infinite plate under the above-mentioned heat transfer conditions can be represented in the form of the following boundary value problem [1]:

$$\frac{\partial \theta(X,\ \mathrm{Fo})}{\partial \mathrm{Fo}} = \frac{\partial^2 \theta(X,\ \mathrm{Fo})}{\partial X^2}, \tag{1}$$

$$\frac{\partial \theta(1,\ \mathrm{Fo})}{\partial X} = \mathrm{Bi}(\mathrm{Fo})\left[\theta_c(\mathrm{Fo}) - \theta(1,\ \mathrm{Fo})\right], \tag{2}$$

$$\frac{\partial \theta(0,\ \mathrm{Fo})}{\partial X} = 0, \tag{3}$$

$$\theta(X,\ 0) = \theta_0, \tag{4}$$

where $\mathrm{Bi(Fo)} = \alpha(\tau)R/\lambda$; $\theta = T/T_{m0}$; T_{m0} is the initial temperature of the external medium; $X = x/R$; $\mathrm{Fo} = a\tau/R^2$. Bi(Fo), θ_m(Fo) are certain differentiable functions.

In [2, 3] quite simple approximate methods are proposed for calculating the temperature field in solids for Bi(Fo) that are valid for somewhat limited region of variation of the Bi number. The present study represents a further development of the methods of solving the heat conduction problem for variable values of the heat transfer coefficient and the temperature of the external medium.

A formal representation of the temperature field can be obtained by reducing boundary value problem (1)–(4) to a functional equation. Solving problem (1)–(4) by means of a Laplace integral transformation, we arrive at an expression for the temperature at any point of the element in terms of its surface value:

$$\theta(X,\ \mathrm{Fo}) = \theta_0 + \int_0^{\mathrm{Fo}} \mathrm{Bi}(\eta)\left[\theta_m(\eta) - \theta(1,\ \eta)\right] d\eta - \mathrm{Bi(Fo)} \times \left[\theta_m(\mathrm{Fo}) - \theta(1,\ \mathrm{Fo})\right]\left(\frac{1-3X^2}{6}\right) + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2}{n^2\pi^2} \cos n\pi X \times \int_0^{\mathrm{Fo}} \left[\mathrm{Bi}(\eta)\{\theta_m(\eta) - \theta(1,\ \eta)\}\right]' \times \exp\left[-n^2\pi^2(\mathrm{Fo}-\eta)\right] d\eta. \tag{5}$$

Finding the general solution of Eq. (5) is very difficult; however, setting $X = 1$ we can derive from it a functional equation for the surface temperature:

$$\theta(1,\ \mathrm{Fo}) = \theta_0 + \int_0^{\mathrm{Fo}} \mathrm{Bi}(\eta)\left[\theta_m(\eta) - \theta(1,\ \eta)\right] d\eta + \frac{1}{3}\mathrm{Bi(Fo)} \times \left[\theta_m(\mathrm{Fo}) - \theta(1,\ \mathrm{Fo})\right] + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2} \int_0^{\mathrm{Fo}} \left[\mathrm{Bi}(\eta)\{\theta_m(\eta) - \theta(1,\ \eta)\}\right]' \times \exp\left[-n^2\pi^2(\mathrm{Fo}-\eta)\right] d\eta. \tag{6}$$

It can be shown that in solving Eq. (6) the process of successive approximations converges [4]. Hence the problem is to find the best first approximation.

As Luikov has shown [5, 6], when heated by a variable heat flux, starting from some Fo > Fo*, a body enters the quasi-stationary stage. From the mathematical standpoint, this indicates that the sum of the series does not have an important influence on the temperature field at Fo > Fo*.

In the stage corresponding to the regular regime of the second kind the solution of Eq. (6) is given by

$$\theta(1,\ \mathrm{Fo}) = \exp\left[-\int_0^{\mathrm{Fo}} \frac{\mathrm{Bi}'(\eta) + 3\mathrm{Bi}(\eta)}{3 + \mathrm{Bi}(\eta)} d\eta\right] \times \left\{\frac{\theta_0 + \frac{1}{3}\mathrm{Bi}(0)\theta_m(0)}{1 + \frac{1}{3}\mathrm{Bi}(0)\theta_m(0)} + \int_0^{\mathrm{Fo}} \frac{3\mathrm{Bi}(\eta)\theta_m(\eta) + \left[\mathrm{Bi}(\eta)\theta_m(\eta)\right]'}{3 + \mathrm{Bi}(\eta)} \times \exp\left[\int_0^{\eta} \frac{\mathrm{Bi}'(\xi) + 3\mathrm{Bi}(\xi)}{3 + \mathrm{Bi}(\xi)} d\xi\right] d\eta\right\}.$$

Temperature Field of an Infinite Plate for $\theta_0 = 0.15$

Fo	θ surface		δ, %	θ center		δ, %
	computer	calc.		computer	calc.	
0.4	0.48228	0.45768	5.1	0.28537	0.27483	3.6
0.5	0.53581	0.52282	2.42	0.33261	0.32562	2.15
0.6	0.57813	0.56714	1.89	0.36663	0.35992	1.87
0.7	0.62749	0.61785	1.54	0.41221	0.40633	1.44
0.8	0.66933	0.66100	1.2	0.44817	0.44391	0.96
0.9	0.71581	0.70883	0.97	0.49873	0.49475	0.81
1.0	0.75512	0.75101	0.545	0.52662	0.52382	0.535
1.1	0.81232	0.80817	0.513	0.60754	0.60611	0.236
1.2	0.84873	0.84623	0.295	0.64612	0.64612	+0.00
1.3	0.88637	0.88637	+0.00	0.69331	0.69331	+0.00
1.4	0.92243	0.92243	+0.00	0.74222	0.74222	+0.00
1.5	0.95561	0.95561	+0.00	0.77013	0.77013	+0.00
1.6	0.98031	0.98031	+0.00	0.79474	0.79474	+0.00
1.7	0.99984	0.99984	+0.00	0.82008	0.82008	+0.00

Substituting the formula obtained for $\theta(1, \mathrm{Fo})$ into (5), we obtain the temperature distribution for an arbitrary section.

To estimate the error of the proposed method, system (1)–(4) was numerically integrated by the Vanichev method [7] on a Minsk-1 computer (table).

As an example we investigated the heating of an infinite plate when the Bi number varies according to the law $\mathrm{Bi(Fo)} = 0.5e^{\mathrm{Fo}}$ and the temperature of the external medium according to the law $\theta_m(\mathrm{Fo}) = 1 + 0.075\mathrm{Fo}$.

The results of a comparison of the data obtained indicate regularization of the heating kinetics under conditions of variable heat transfer and variable external temperature. The relations obtained can be used in investigating processes of heat propagation under the conditions $\mathrm{Bi} = \mathrm{Bi(Fo)}$ and $\theta_m = \theta_m(\mathrm{Fo})$.

The proposed method can be extended successfully to classical bodies of different geometry.

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